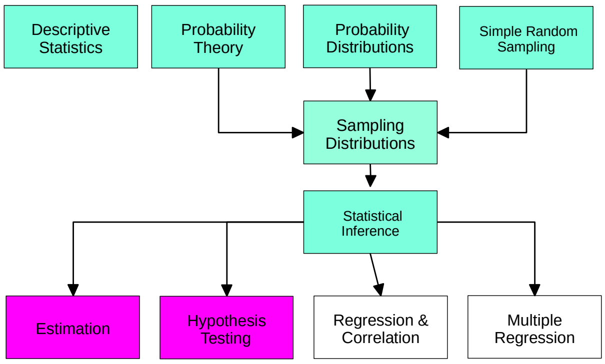
The Course So Far:



Hypothesis Testing: Preliminaries

- A **hypothesis** is a statement that something is true.
- **Null hypothesis:** A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis
- **Alternative hypothesis:** A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

In this course, we will always assume that the null hypothsis for a population parameter, Θ, always specifies a single value for that parameter. So, an equal sign always appears:

$$H_0 : \Theta = \Theta_0$$

If the primary concern is deciding whether a population parameter is *different than* a specified value, the alternative hypothesis should be:

$$H_a : \Theta \neq \Theta_0$$

This form of alternative hypothesis is called a **two-tailed test.**

If the primary concern is whether a population parameter, Θ, is *less than* a specified value Θ_0, the alternative hypothesis should be:

$$H_a : \Theta < \Theta_0$$

A hypothesis test whose alternative hypothesis has this form is called a **left-tailed test.**

If the primary concern is whether a population parameter, Θ, is *greater than* a specified value Θ_0, the alternative hypothesis should be:

$$H_a : \Theta > \Theta_0$$

A hypothesis test whose alternative hypothesis has this form is called a **right-tailed test.**

A hypothesis test is called a **one-tailed test** if it is either right- or left-tailed, i.e., if it is not a two-tailed test.

Weight
465
456
438
454
447
449
442
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468
433
454
463
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446
447
456
452
444
447
456
456
435
450

```
. use "D:\Amifiles\Econ 70\lectures_new\pretzels.dta", clear

. summarize weight, detail

                          weight
-------------------------------------------------------------
      Percentiles      Smallest
 1%          433            433
 5%          435            435
10%          438            438       Obs                  25
25%          446            442       Sum of Wgt.          25

50%          449                      Mean                450
                        Largest       Std. Dev.      8.480369
75%          456            456
90%          463            463       Variance       71.91667
95%          465            465       Skewness       .0991536
99%          468            468       Kurtosis       2.945427
```

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2}$$

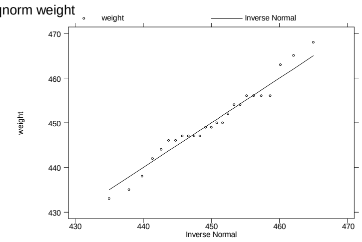
$$x = f^{-1}(x) = f^{-1}\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2}\right)$$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$t_{\alpha/2} = t_{.025} = 2.064$$

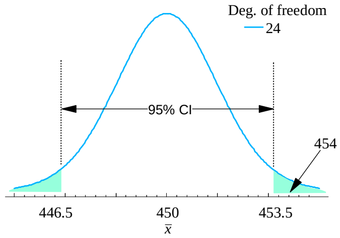
$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 450 \pm 2.064\left(\frac{8.480369}{5}\right) = 450 \pm 3.50$$

qnorm weight



The 95% confidence interval is 446.5 -- 453.5

Confidence Interval Centered on Sample Mean



t

- State the null and alternative hypotheses:

$$H_0 : \mu = 454$$
$$H_a : \mu \neq 454$$

- Decide on the significance level, α:

$$\alpha = 0.05$$

- Compute the value of the test statistic, t:

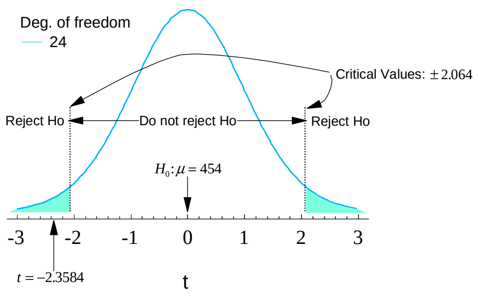
$$t = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{450 - 454}{\frac{8.480369}{\sqrt{25}}} = \frac{-4}{1.69607} = -2.3584$$

- Determine the critical value(s). We're looking for the t-values that will put 2.5% of the area in *each* tail:

± 2.064 as before, when we calculated the confidence interval

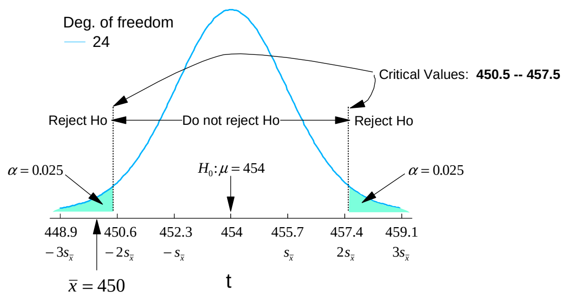
Hypothesis Test: Centered on Ho, mean = 454

t-distribution with standardized *t-value* on horizonal axis



Hypothesis Test: Centered on Ho, mean = 454

t-distribution with unstandardized *sample mean values* on horizonal axis

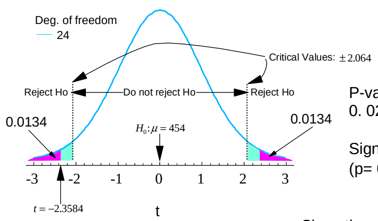


Two approaches to hypothesis testing

Critical Value Approach	P- Value Approach
Step 1: State the null and alternative hypotheses	Step 1: State the null and alternative hypotheses
Step 2: Decide on the significance level, α .	Step 2: Decide on the significance level, α .
Step 3: Compute the value of the test statistic	Step 3: Compute the value of the test statistic
Step 4: Determine the critical value(s)	Step 4: Determine the *P*-value
Step 5: If the value of the test statistic falls in the rejection region, reject Ho; otherwise, do not reject Ho.	Step 5: If $P \le \alpha$, reject Ho; otherwise, do not reject Ho.
Step 6: Interpret the result of the hypothesis test.	Step 6: Interpret the result of the hypothesis test.

Definition: The *P*-value of a hypothesis test equals the smallest significance level at which the null hypothesis can be rejected, that is, the smallest level for which the observed sampled data results in rejection of Ho.

Decision Criterion: If the *P*-value is less than or equal to the specified significance level, reject the null hypothesis; otherwise, do not reject the null hypothesis.



P-value = 0.0134 + 0.0134 = 0. 0268

Significance level, α , = 0.05 (p= 0.025 in each tail (blue)).

Since the *p*-value is less than the chosen significance level, we can reject Ho.

One-tailed tests

Significance level, $\alpha = 0.05$

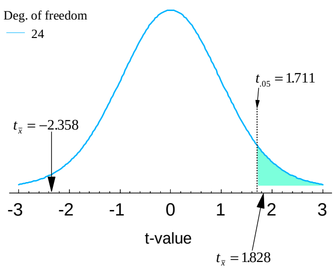
$H_0: \mu = 454$

$H_a: \mu > 454$

Compute the *t*-value from the sample mean:

$$t = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{450 - 454}{\frac{8.48037}{5}} = -2.358$$

One-tailed hypothesis test: t-distribution



We cannot reject the null hypothesis that $\mu = 454$

What if $\bar{x} = 457.1$?

$$t = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{457.1 - 454}{\frac{8.48037}{5}} = 1.828$$

In this case we <u>can</u> reject the null hypothesis that $\mu = 454$

Left-tailed test:

Significance level, $\alpha = 0.05$

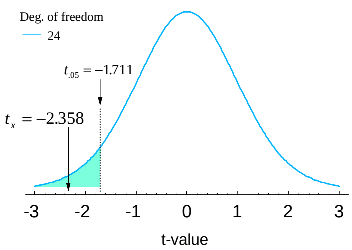
$H_0: \mu = 454$

$H_a: \mu < 454$

$$\bar{x} = 450$$

$$t = \frac{\bar{x} - \mu_{H_0}}{\frac{s}{\sqrt{n}}} = \frac{450 - 454}{\frac{8.48037}{5}} = -2.358$$

Left-tailed hypothesis test: t-distribution



In this case we <u>can</u> reject the null hypothesis that $\mu = 454$

the *p-value* is *0.0134*

If we drew another sample and found a sample mean > 454, what could we say about the null hypothesis?

Immediately, we can say that it is <u>not</u> possible to reject the null hypothesis.T

Type I and Type II Errors and the Power of a Test

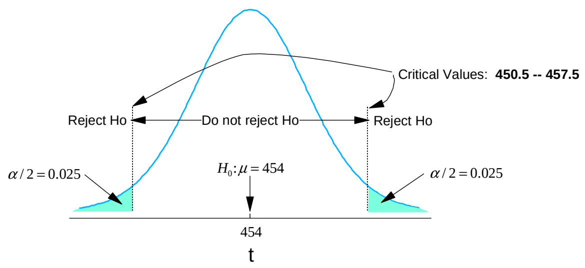
Type I error: rejection of a true null hypothesis
Type II error: non rejection (acceptance) of a false null hypothesis

		H0 is:	
		True	False
Decision	Do not Reject H0	Correct Decision	Type II Error
	Reject H0	Type I error	Correct Decision

Pretzel Bags

Hypothesis Test: Centered on Ho, mean = 454

Assume that the true population mean actually = 454



The probability of making a Type I error is the probability of having a sample mean in the shaded tails of the sampling distribution centered on Ho. This is the *significance level* of the test, and is denoted by α.

Key Fact:

For a fixed sample size, the smaller we specify the significance level, α the larger will be the probability of a Type II error, β of not rejecting a false null hypothesis.

Type II error probabilities for various true means, sample size = 30

$$H_0 : \mu = 26.0$$
$$H_a : \mu \le 26.0$$

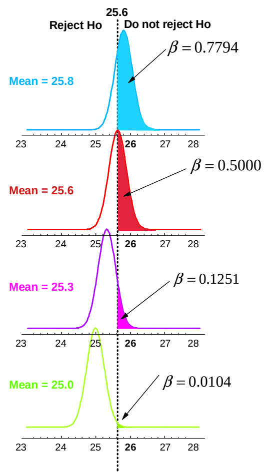
Let: $\alpha = 0.05, \quad \sigma = 1.4\ mpg$

This implies:

$$z_{0.05} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 26}{1.4/\sqrt{30}} \to \bar{x} = 26 - 1.645 \cdot \left(\frac{1.4}{\sqrt{30}}\right) = 25.6$$

The critical value for rejection is: $\bar{x} = 25.6$

Depending upon the true value of the mean, the probability of incorrectly failing to reject the null hypothesis, i.e., making a Type II error may be:



The closer the true mean is to the hypothesized null value, the higher the probability of making a Type II error, for a given and constant sample size.

To summarize: part of evaluating the effectiveness of a hypothesis test involves an analysis of the chances of making an incorrect decision.

1. The probability of making a **Type I error** is specified by the significance level, α .
2. The probability of making a **Type II error** depends on the true value of the parameter in question.

Statisticians refer to the probability of **not** making a Type II error (i.e., the probability of rejecting a false null hypothesis) as the **power of the test:**

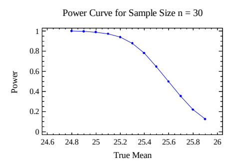
$$\text{Power} = 1 - P(\text{Type II error}) = 1 - \beta.$$

The power of a hypothesis test is between 0 and 1 and measures the ability of the hypothesis test to detect a false null hypothesis:

Power $\approx 0 \Rightarrow$ test is not very good at detecting a false null hypothesis.
Power $\approx 1 \Rightarrow$ test is extremely good at detecting a false null hypothesis

Power table for Humvee Mini mileage:

True mean: μ	$P(\text{Type II error}) = \beta$	Power $= 1-\beta$
25.9	0.8747	0.1251
25.8	0.7794	0.2206
25.7	0.6480	0.3520
25.6	0.5000	0.5000
25.5	0.3520	0.6480
25.4	0.2206	0.7794
25.3	0.1251	0.8784
25.2	0.0618	0.9382
25.1	0.0274	0.9726
25.0	0.0104	0.9896
24.9	0.0036	0.9964
24.8	0.0010	0.9990



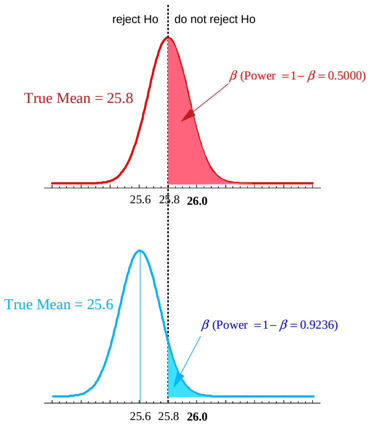
$$H_0 : \mu = 26.0$$
$$H_a : \mu \le 26.0$$

Let: $\alpha = 0.05, \quad \sigma = 1.4\ mpg$

This implies:

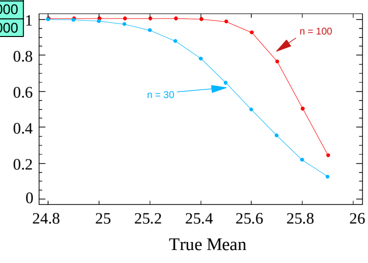
$$z_{0.05} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 26}{1.4/\sqrt{100}} \to \bar{x} = 26 - 1.645 \cdot \left(\frac{1.4}{\sqrt{100}}\right) = 25.8$$

25.8 is the critical value at which we reject the null hypothesis that mileage = 26.0 mpg.



True mean: μ	$P(\text{Type II error})=\beta$	Power $=1-\beta$
25.9	0.7611	0.2389
25.8	0.5000	0.5000
25.7	0.2389	0.7611
25.6	0.0764	0.9236
25.5	0.0162	0.9838
25.4	0.0021	0.9979
25.3	0.0002	0.9998
25.2	0.0000	1.0000
25.1	0.0000	1.0000
25.0	0.0000	1.0000
24.9	0.0000	1.0000
24.8	0.0000	1.0000

Power Curves for n = 30 and n = 100



The Moral of the Story: For a fixed significance level, increasing the sample size increases the power.

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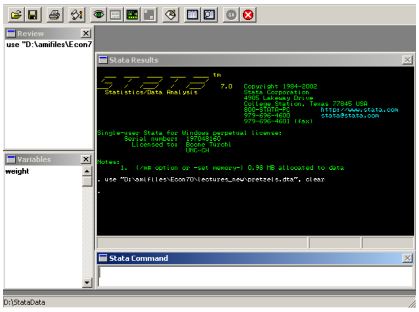
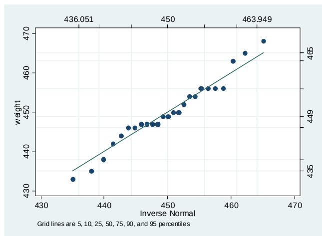


qnorm weight,grid

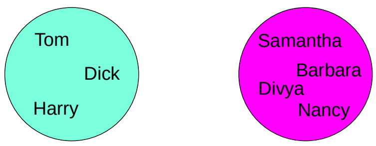



```
ttest  weight=454
```

```
One-sample t test

------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  weight |      25         450    1.696074    8.480369    446.4995    453.5005
------------------------------------------------------------------------------
Degrees of freedom: 24

                           Ho: mean(weight) = 454

    Ha: mean < 454             Ha: mean ~= 454             Ha: mean > 454
      t =  -2.3584               t =  -2.3584               t =  -2.3584
  P < t =   0.0134           P > |t| =   0.0268         P > t =   0.9866
```

Male Population

Female Population



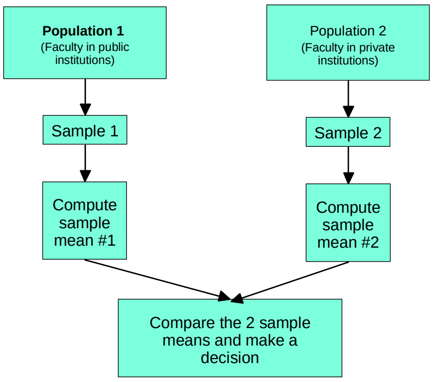
Males: $\binom{3}{2}=3$ samples

Females: $\binom{4}{3}=4$ samples

$3\times 4=12$ possible pairs of samples

If each of the 12 has an equal likelihood of being selected (i.e., 1/12 in this case) then the samples are *independent.*

Process for comparing 2 population means, using independent samples



Population 1: Faculty in public institutions
Population 2: Faculty in private institutions

μ_1 = mean salary of faculty in public institutions;
μ_2 = mean salary of faculty in private institutions.

$H_0: \mu_1 = \mu_2$ (mean salaries are the same)
$H_a: \mu_1 \neq \mu_2$ (mean salaries are different)

Sample 1: Salaries of faculty members in public institutions (n=30)

34.2 63.6 24.4 79.4 33.8 88.2 90.0 56.8 56.0 42.2 40.2 44.6 100.4 41.4 58.2 81.8
51.2 64.4 24.6 35.0 76.8 29.2 41.2 74.0 107.4 54.2 84.2 15.8 60.2 71.0

Sample 2: Salaries of faculty members in private institutions (n=35)

92.9 102.2 51.5 77.6 71.1 59.3 71.0 52.0 62.9 46.4 61.6 73.5 97.5 97.3 63.1 53.8
45.2 78.3 67.6 27.2 92.6 118.5 101.0 76.0 66.3 52.4 81.2 56.0 37.7 68.6 56.1
31.1 47.2 24.8 62.3

Means of Samples 1 & 2:

$$\bar{x}_1 = \frac{\sum x}{n_1} = \frac{1724.4}{30} = 57.48$$

$$\bar{x}_2 = \frac{\sum x}{n_2} = \frac{2323.8}{35} = 66.39$$

Can the difference, -8.914 ($8,914) between these two means be reasonably attributed to sampling error, or is it large enough to conclude that the two populations have different means?

```
. sort type

. by  type: summarize  salary,  detail

_______________________________________________________________________________
-> type = Public

                           salary
-------------------------------------------------------------
      Percentiles      Smallest
 1%         15.8           15.8
 5%         24.4           24.4
10%         26.9           24.6       Obs                  30
25%         40.2           29.2       Sum of Wgt.          30

50%         56.4                      Mean              57.48
                        Largest       Std. Dev.       23.9528
75%         76.8           88.2
90%         89.1             90       Variance       573.7368
95%        100.4          100.4       Skewness       .2602229
99%        107.4          107.4       Kurtosis       2.183742

_______________________________________________________________________________
-> type = Private

                           salary
-------------------------------------------------------------
      Percentiles      Smallest
 1%         24.8           24.8
 5%         27.2           27.2
10%         37.7           31.1       Obs                  35
25%           52           37.7       Sum of Wgt.          35

50%         63.1                      Mean           66.39429
                        Largest       Std. Dev.      22.26112
75%         78.3           97.5
90%         97.5            101       Variance       495.5576
95%        102.2          102.2       Skewness       .2560263
99%        118.5          118.5       Kurtosis       2.665608
```

Descriptive Statistics: Faculty Salaries
Public and Private Universities

The Sampling Distribution of the Difference Between Two Sample Means for Independent Samples:

Suppose that x is a normally distributed variable on each of two populations. Then, for independent samples of sizes n_1 and n_2 from the two populations,

- $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2,$
- $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}$, and
- $\bar{x}_1 - \bar{x}_2$ is normally distributed.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$$

Assuming equal population variances, we get:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma\sqrt{(1/n_1) + (1/n_2)}}$$

The Pooled Sample Variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The Pooled Sample Standard Deviation:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$ the pooled sample standard deviation

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma\sqrt{(1/n_1) + (1/n_2)}} \rightarrow t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{(1/n_1) + (1/n_2)}}$$ The pooled t-statistic

Distribution of the Pooled t-statistic
Suppose that x is a normally distributed variable on each of two populations and that the population standard deviations are equal. Then, for independent samples of sizes n1 and n2 from the two populations, the variable

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{(1/n_1) + (1/n_2)}}$$

has the t-distribution with $df = n_1 + n_2 - 2$.

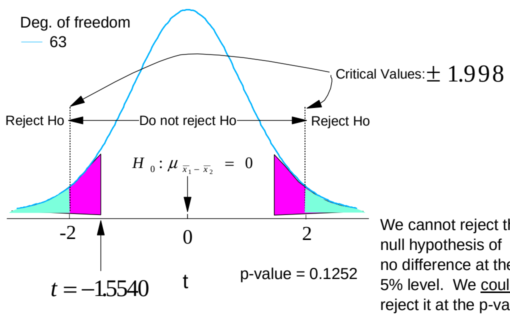
$H_0: \mu_1 = \mu_2$ (mean salaries are the same)
$H_a: \mu_1 \neq \mu_2$ (mean salaries are different)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{(1/n_1) + (1/n_2)}} = \frac{(57.48 - 66.39) - (\mu_1 - \mu_2 = 0)}{23.05\sqrt{(1/30) + (1/35)}} = -1.554$$

where $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(30-1)\cdot(23.95)^2 + (35-1)\cdot(22.26)^2}{30 + 35 - 2}} = 23.05$

Hypothesis Test: Centered on Ho, difference between means = 0

t-distribution with standardized *t-value* on horizonal axis

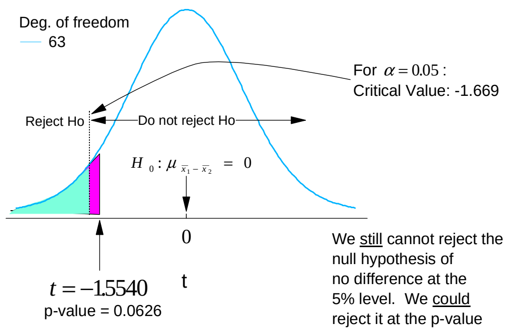


We cannot reject the null hypothesis of no difference at the 5% level. We <u>could</u> reject it at the p-value = 12.52%.

Hypothesis Test: Centered on Ho, difference between means = 0

Ha: $\mu_1 < \mu_2$

t-distribution with standardized *t-value* on horizonal axis



We <u>still</u> cannot reject the null hypothesis of no difference at the 5% level. We <u>could</u> reject it at the p-value = 6.26%.

. ttest salary, by(type)

```
Two-sample t test with equal variances

------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  Public |      30       57.48    4.373164     23.9528    48.53588    66.42412
 Private |      35    66.39429    3.762817    22.26112    58.74732    74.04125
---------+--------------------------------------------------------------------
combined |      65       62.28    2.891091    23.30872    56.50438    68.05562
---------+--------------------------------------------------------------------
    diff |           -8.914285    5.736302               -20.37737    2.548799
------------------------------------------------------------------------------
Degrees of freedom: 63

                 Ho: mean(Public) - mean(Private) = diff = 0

     Ha: diff < 0               Ha: diff ~= 0              Ha: diff > 0
       t =  -1.5540               t =  -1.5540              t =  -1.5540
   P < t =   0.0626         P > |t| =   0.1252          P > t =   0.9374

. save "D:\amifiles\Econ70\lectures_new\Fac_Salary.dta"
file D:\amifiles\Econ70\lectures_new\Fac_Salary.dta saved
```

Prove: $\bar{x}_1 - \bar{x}_2$ is an unbiased estimator of $\mu_1 - \mu_2$

$E[\bar{x}_1 - \bar{x}_2] = E[\bar{x}_1] - E[\bar{x}_2]$ (algebra of expectations)

$$E[\bar{x}_1] = E\left[\frac{\sum_{i=1}^{n} x_i^1}{n_1}\right] = \frac{1}{n_1} \times E\left[\sum_{i=1}^{n} x_i^1\right] = \frac{1}{n_1} \times n_1 \times E\left[x^1\right] = \mu_1$$

and

$$E[\bar{x}_2] = E\left[\frac{\sum_{i=1}^{n} x_i^2}{n_2}\right] = \frac{1}{n_2} \times E\left[\sum_{i=1}^{n} x_i^2\right] = \frac{1}{n_2} \times n_2 \times E\left[x^2\right] = \mu_2$$

$$\therefore E[\bar{x}_1 - \bar{x}_2] = E[\bar{x}_1] - E[\bar{x}_2] = \mu_1 - \mu_2$$

And, assuming the populations have equal variances, our pooled estimator of the population variance is (as before):

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(30-1)\cdot(23.95)^2 + (35-1)\cdot(22.26)^2}{30+35-2}} = 23.30872$$

And the standard error of the difference between the means is:

$$s_{\bar{x}_1-\bar{x}_2} = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 23.30872\sqrt{0.0333+0.02857} = 23.30872\sqrt{0.06190} = 23.30872 \times 0.24881 = 5.79937$$

So, we can write the 95% confidence interval (with *df*= n_1+n_2-2) as:

$$\bar{x}_1 - \bar{x}_2 \pm t_{\frac{\alpha}{2}} \times s_{\bar{x}_1-\bar{x}_2} = -8.914285 \pm 1.96 \times 5.79937$$

or, $(-20.3, \quad 2.5)$

Which is just what Stata gave us:

$\bar{x}_1 - \bar{x}_2$

S_p

```
Two-sample t test with equal variances

------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  Public |      30       57.48    4.373164     23.9528    48.53588    66.42412
 Private |      35    66.39429    3.762817    22.26112    58.74732    74.04125
---------+--------------------------------------------------------------------
combined |      65       62.28    2.891091    23.30872    56.50438    68.05562
---------+--------------------------------------------------------------------
    diff |           -8.914285    5.736302               -20.37737    2.548799
------------------------------------------------------------------------------
Degrees of freedom: 63

                 Ho: mean(Public) - mean(Private) = diff = 0

     Ha: diff < 0               Ha: diff ~= 0              Ha: diff > 0
       t =  -1.5540               t =  -1.5540              t =  -1.5540
   P < t =   0.0626         P > |t| =   0.1252          P > t =   0.9374

. save "D:\amifiles\Econ70\lectures_new\Fac_Salary.dta"
file D:\amifiles\Econ70\lectures_new\Fac_Salary.dta saved
```

$S_{\bar{x}_1-\bar{x}_2}$

Confidence Interval

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$$

Calculation of degrees of freedom:

$$\Delta = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$ rounded down to the nearest integer.

ttest salary,by(type) une

```
Two-sample t test with unequal variances

------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  Public |      30       57.48    4.373164     23.9528    48.53588    66.42412
 Private |      35    66.39429    3.762817    22.26112    58.74732    74.04125
---------+--------------------------------------------------------------------
combined |      65       62.28    2.891091    23.30872    56.50438    68.05562
---------+--------------------------------------------------------------------
    diff |           -8.914285    5.769172               -20.45493    2.626359
------------------------------------------------------------------------------
Satterthwaite's degrees of freedom:  59.8534

                 Ho: mean(Public) - mean(Private) = diff = 0

     Ha: diff < 0               Ha: diff ~= 0              Ha: diff > 0
       t =  -1.5452               t =  -1.5452               t =  -1.5452
   P < t =   0.0638           P > |t| =   0.1276          P > t =   0.9362
```

Definition: With a **random paired sample**, each possible paired sample is equally likely to be the one selected.

Husband's and Wife's Report of Husband's Weekly Housework.

Couple Number	diffference	Husband's Report	Wife's Report
1	5.8	19.8	14.0
2	10.0	19.2	9.2
3	3.1	18.3	15.2
4	0.7	16.9	16.2
5	0.3	16.3	16.0
6	-0.2	16.0	16.2
7	2.3	14.8	12.5
8	-0.8	14.6	15.4
9	0.6	14.0	13.4
10	-2.1	12.7	14.8
11	-1.8	11.6	13.4
12	-3.8	11.5	15.3

```
. summarize  hwdiff,detail

                           hwdiff
-------------------------------------------------------------
      Percentiles      Smallest
 1%         -3.8           -3.8
 5%         -3.8           -2.1
10%         -2.1           -1.8       Obs                  12
25%         -1.3            -.8       Sum of Wgt.          12

50%          .45                      Mean              1.175
                        Largest       Std. Dev.      3.762041
75%          2.7            2.3
90%          5.8            3.1       Variance       14.15295
95%           10            5.8       Skewness       1.068076
99%           10             10       Kurtosis       3.645884
```

$$H_0 : \mu_d = 0 \quad where\ \mu_d = \mu_1 - \mu_2$$

$$H_a : \mu_d \neq 0\ or\ \mu_d > 0$$

$$t = \frac{\bar{d} - (\mu_1 - \mu_2)}{s_d / \sqrt{n}}$$

where $\bar{d} = \frac{\sum_{i=1}^{n} (x_i^h - x_i^w)}{n}$ and $s_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$

```
. ttest hwdiff=0

One-sample t test

------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  hwdiff |      12       1.175    1.086008    3.762041   -1.215287    3.565287
------------------------------------------------------------------------------
Degrees of freedom: 11

                          Ho: mean(hwdiff) = 0

     Ha: mean < 0             Ha: mean != 0             Ha: mean > 0
       t =   1.0819             t =   1.0819             t =   1.0819
   P < t =   0.8488         P > |t| =   0.3024         P > t =   0.1512
```

Two-tail test

Right-tail test

Can you write the expression for the 95% confidence interval?

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$H_0 : p = p_0$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Assume both np_0 and $n(1-p_0)$ are 5 or greater

Step 1: The null hypothesis is $H_0 : p = p_0$ and the alternative hypothesis is:

$H_a : p \neq p_0$ (Two-tailed) or $H_a : p < p_0$ (Left-tailed) or $H_a : p > p_0$ (Right-tailed)

Step 2: Decide on the significance level α

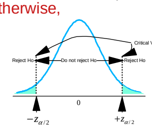
Step 3: Compute the value of the test-statistic

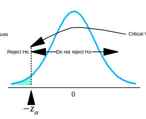
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

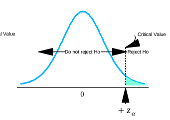
Step 4: The critical value(s) are

$\pm z_{\alpha/2}$ (Two-tailed) or $-z_{\alpha}$ (Left-tailed) or $+z_{\alpha}$ (Right-tailed)

Step 5: If the value of the test statistic falls in the rejection region, reject the null hypothesis; otherwise, do not reject.







n = 1,250 *p*o *= 0.50*

$np_0 = 1250 \bullet 0.50 = 625$

$n(1-p_0) = 1250 \bullet (1-0.50) = 625$

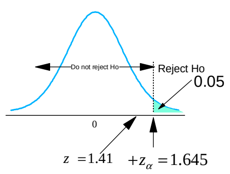
Both are greater than 5 so we can use the normal approximation.

$H_0 : p = 0.50$ (it is not true that a majority favor a ban)

$H_\alpha : p > 0.50$ (a majority favor the ban)

Let the significance level, $\alpha = 0.05$

The value of the test statistic is: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.520 - 0.50}{\sqrt{\frac{0.50 \bullet (1-0.50)}{1250}}} = 1.41$



It appears that we cannot reject the null hypothesis in favor of the alternative.

Population 1: All U.S. men
Population 2: All U.S. women

p_1 and p_2 are the population proportions for the two populations

$H_0 : p_1 = p_2$ (percentage for men is not less than that for women)

$H_a : p_1 < p_2$ (percentage for men is less than that for women)

- Compute the proportion of the men sampled who sometimes order veg, $\hat{p}_1$ and the proportion of women sampled who sometimes order veg, $\hat{p}_2$
- If $\hat{p}_1$ is too much smaller than $\hat{p}_2$ reject null hypothesis; otherwise, do not reject it.

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{276}{747} = 0.369\,(36.9\%)$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{195}{434} = 0.449\,(44.9\%)$$

Key Fact:
For independent samples of sizes n_1 and n_2 from two populations,

- $\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$
- $\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
- $\hat{p}_1 - \hat{p}_2$ is approximately normally distributed for large n_1 and n_2.

In particular, for large samples, the possible differences between the two sample proportions have approximately a normal distribution with mean $p_1 - p_2$ and standard deviation

$$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

and, if the null hypothesis is true, then

$$p_1 - p_2 = 0$$

and we can write:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{p(1-p)}{n_1} + \frac{p(1-p)}{n_2}}} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)} \bullet \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where *p* is the (unknown) common value of the two population proportions.

Our best estimate of *p* is obtained by pooling the sample proportions to produce a **pooled sample proportion**:

$$\hat{p}_p = \frac{x_1 + x_2}{n_1 + n_2}$$

Our *z-statistic* for a **two-sample *z*-test** is therefore:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p(1-\hat{p}_p)} \bullet \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Assume:
1. Independent Samples
2. $x_1, n_1 - x_1, x_2, \& \; n_2 - x_2$ are all 5 or greater.

Step 1: The null hypothesis is $H_0 : p_1 = p_2$ and the alternative hypothesis is:

$H_a : p_1 \neq p_2$ (Two-tailed) or $H_a : p_1 < p_2$ (Left-tailed) or $H_a : p_1 > p_2$ (Right-tailed)

Step 2: Decide on the significance level α

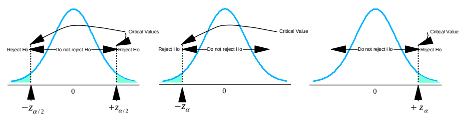
Step 3: Compute the value of the test-statistic

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p(1-\hat{p}_p)} \bullet \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where, $\hat{p}_p = \frac{x_1 + x_2}{n_1 + n_2}$

Step 4: The critical value(s) are

$\pm z_{\alpha/2}$ (Two-tailed) or $-z_{\alpha}$ (Left-tailed) or $+z_{\alpha}$ (Right-tailed)



Population 1: All U.S. men
Population 2: All U.S. women

p_1 and p_2 are the population proportions for the two populations

$H_0 : p_1 = p_2$ (percentage for men is not less than that for women)

$H_a : p_1 < p_2$ (percentage for men is less than that for women)

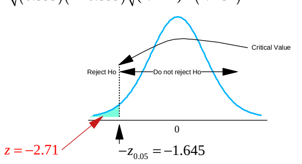
$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{276}{747} = 0.369\,(36.9\%)$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{195}{434} = 0.449\,(44.9\%)$$

$$\hat{p}_p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{276+195}{747+434} = \frac{471}{1181} = 0.399$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p(1-\hat{p}_p)} \bullet \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.369 - 0.449}{\sqrt{(0.399)(1-0.399)}\sqrt{(1/747)+(1/434)}} = -2.71$$

$$-z_{0.05} = -1.645$$



Reject the null hypothesis that the percentage for men is not less than that for women.

$$E[\hat{p}_1 - \hat{p}_2] = E[\hat{p}_1] - E[\hat{p}_2]$$

(The Expectation of a difference equals the difference of expectations)

$$E[\hat{p}_1] = E\left[\frac{x_1}{n_1}\right] = \frac{1}{n_1}E[x_1] = \frac{1}{n_1}n_1 \times p_1 = p_1$$

$$E[\hat{p}_2] = E\left[\frac{x_2}{n_2}\right] = \frac{1}{n_2}E[x_2] = \frac{1}{n_2}n_2 \times p_2 = p_2$$

$$\therefore E[\hat{p}_1 - \hat{p}_2] = p_1 - p_2$$

(The expectation of the difference between sample proportions is equal to the difference between population proportion ... i.e., The difference between sample proportions is an unbiased estimator of the difference between population proportions)

$$\hat{\sigma}_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = .1185209$$

$$\hat{p}_1 - \hat{p}_2 \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = (.2872278, .7518214)$$

```
prtest cylforeign, by( foreign)

Two-sample test of proportion                    0: Number of obs =        31
                                                 1: Number of obs =        19

------------------------------------------------------------------------------
Variable |       Mean    Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |  .6774194     .083959   8.06846   0.0000       .5128628    .8419759
       1 |  .1578947    .0836547   1.88746   0.0591      -.0060654    .3218549
---------+--------------------------------------------------------------------
    diff |  .5195246    .1185209                          .2872278    .7518214
         |  under Ho:   .1455625   3.56908   0.0004
------------------------------------------------------------------------------

              Ho: proportion(0) - proportion(1) = diff = 0

     Ha: diff < 0              Ha: diff ~= 0              Ha: diff > 0
       z =  3.569                z =  3.569                 z =  3.569
   P < z =  0.9998         P > |z| =  0.0004          P > z =  0.0002

.
```

$\hat{p}_0 - \hat{p}_1$

95% Confidence Interval

$$\hat{\sigma}_{(\hat{p}_0 - \hat{p}_1)} = \sqrt{\frac{\hat{p}_0(1-\hat{p}_0)}{n_0} + \frac{\hat{p}_1(1-\hat{p}_1)}{n_1}}$$

$$y_i = \frac{x_i - \mu_i}{\sigma_i}$$

$$u = \sum_{i=1}^{k} y_i^2 = \sum_{i=1}^{k} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2$$

The random variable u is the sum of k squared standard normal random variables. It is called the **chi-square distribution** with k degrees of freedom (one df for each of the terms in the sum).

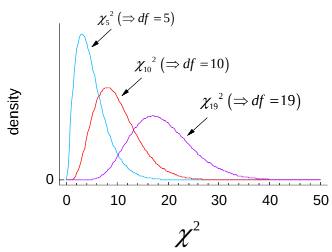
Density function for the chi-square distribution:

$$f(u;k) = \frac{1}{[(k/2)-1]!\,2^{k/2}} u^{(k/2)-1} e^{-(1/2)u}, \quad u > 0$$

The random variable u is usually designated by the Greek letter Chi, squared: χ^2

$$\chi^2_{(k)} = \sum_{i=1}^{k} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2 \qquad \mu_{\chi^2_{(k)}} = k \qquad \sigma_{\chi^2_{(k)}} = 2k$$

Chi-Square Distribution



Basic properties of χ^2-curves:

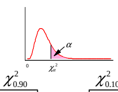
Property 1: The total area under a χ^2 curve equals 1.

Property 2: A χ^2-curve starts at zero on the horizontal axis and extends indefinitely to the right, approaching, but never touching, the horizontal axis as it does so.

Property 3: A χ^2-curve is right skewed.

Property 4: As the number of degrees of freedom becomes larger, χ^2-curves look increasingly like normal curves.

Critical values of χ^2_α



df	$\chi^2_{0.995}$	$\chi^2_{0.99}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.90}$	$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.001}$	$\chi^2_{0.005}$	df
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	1
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	2
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	3
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	4
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	5
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	6
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	15
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	16
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	17
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	18
19	6.844	7.633	8.907	10.117	11.651	27.204	30.143	32.852	36.191	38.582	19
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	20
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	21
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.290	42.796	22
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	23
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	24
25	10.520	11.524	13.120	14.611	16.473	34.382	37.653	40.647	44.314	46.928	25
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	26
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	27
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994	28
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	29
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	30
40	20.707	22.164	24.433	26.509	29.051	51.805	55.759	59.342	63.691	66.767	40
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	50
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.381	91.955	60
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.424	104.213	70
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.628	112.328	116.320	80
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.135	124.115	128.296	90
100	67.328	70.065	74.222	77.930	82.358	118.499	124.343	129.563	135.811	140.177	100

Handout on the Chi-square Distribution

$$y_i = \frac{x_i - \mu_i}{\sigma_i}$$

$$u = \sum_{i=1}^{k} y_i^2 = \sum_{i=1}^{k} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2$$

The random variable u is the sum of k squared standard normal random variables. It is called the **chi-square distribution** with k degrees of freedom (one df for each of the terms in the sum).

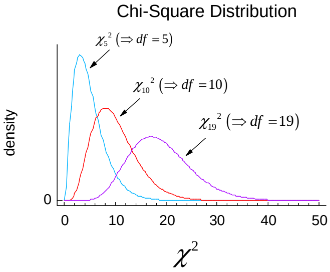
Density function for the chi-square distribution:

$$f(u;k) = \frac{1}{[(k/2)-1]!\,2^{k/2}} u^{(k/2)-1} e^{-(1/2)u}, \quad u > 0$$

The random variable u is usually designated by the Greek letter Chi, squared: χ^2

$$\chi^2_{(k)} = \sum_{i=1}^{k} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2 \qquad \mu_{\chi^2_{(k)}} = k \qquad \sigma_{\chi^2_{(k)}} = 2k$$

Chi-Square Distribution



Basic properties of χ^2-curves:

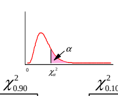
Property 1: The total area under a χ^2 curve equals 1.

Property 2: A χ^2-curve starts at zero on the horizontal axis and extends indefinitely to the right, approaching, but never touching, the horizontal axis as it does so.

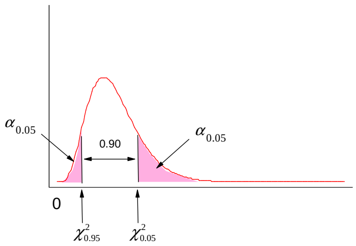
Property 3: A χ^2-curve is right skewed.

Property 4: As the number of degrees of freedom becomes larger, χ^2-curves look increasingly like normal curves.

Critical values of χ^2_α



df	$\chi^2_{0.995}$	$\chi^2_{0.99}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.90}$	$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.001}$	$\chi^2_{0.005}$	df
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	1
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	2
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	3
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	4
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	5
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	6
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	15
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	16
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	17
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	18
19	6.844	7.633	8.907	10.117	11.651	27.204	30.143	32.852	36.191	38.582	19
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	20
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	21
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.290	42.796	22
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	23
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	24
25	10.520	11.524	13.120	14.611	16.473	34.382	37.653	40.647	44.314	46.928	25
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	26
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	27
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994	28
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	29
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	30
40	20.707	22.164	24.433	26.509	29.051	51.805	55.759	59.342	63.691	66.767	40
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	50
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.381	91.955	60
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.424	104.213	70
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.628	112.328	116.320	80
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.135	124.115	128.296	90
100	67.328	70.065	74.222	77.930	82.358	118.499	124.343	129.563	135.811	140.177	100



The *F*-distribution:

$$Y = \frac{U/m}{V/n}$$

The *F*-distribution is formed as the ratio of two chi-square variates divided by their respective degrees of freedom

Density function for the *F*-distribution

$$f(y) = \frac{\Gamma\left[(m+n)/2\right]}{\Gamma(m/2)\Gamma(n/2)}\left(\frac{m}{n}\right)^{m/2}\left(1+\frac{m}{n}y\right)^{(m+n)/2} y^{(m-2)/2}$$

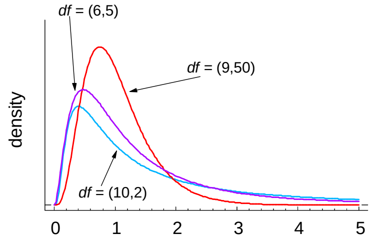
The F-distribution:

- Depends upon two parameters, *m* and *n* which are, respectively the number of degrees of freedom of the chi-square variates in the numerator and denominator.
- Total area under the *F*-curve =1.
- Is skewed to the right.
- Has a range from zero to infinity.
- Changes shape as the numbers of the degrees of freedom change.
- As *m* and *n* become large the *F*-distribution has the normal distribution as its limit.
- The mean and variance of the distribution are:

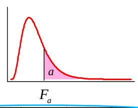
$$\mu = \frac{n}{n-2}, \quad n>2 \qquad \sigma^2 = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}, \quad n>4$$

The variance does not exist for *n* less than or equal to 4 and the mean does not exist for *n* less than or equal to 2.

F (variance ratio) Distribution



df = (numerator df, denominator df)



Percentage Points of the F-Distribution

		V1							
V2	a	1	2	3	4	5	6	7	8
1	0.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44
	0.050	161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90
	0.025	647.80	799.50	864.20	899.60	921.80	937.10	948.20	956.70
	0.010	4052.00	4999.50	5403.00	5625.00	5764.00	5859.00	5928.00	5982.00
	0.005	16211.00	20000.00	21615.00	22500.00	23056.00	23437.00	23715.00	23925.00
2	0.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37
	0.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37
	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37
	0.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37
	0.005	198.50	199.00	199.20	199.20	199.30	199.30	199.40	199.40
3	0.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25
	0.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85
	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54
	0.010	34.12	30.82	29.46	28.71	28.24	27.91	27.64	27.49
	0.005	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13
4	0.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95
	0.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04
	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98
	0.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80
	0.005	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35

Key Fact: For an *F*-curve with $df = (\nu_1, \nu_2)$, the *F*- value having area α to its left equals the reciprocal of the *F*-value having area α to its right for an *F*-curve with $df = (\nu_2, \nu_1)$.

The *F*-distribution:

$$Y = \frac{U/m}{V/n}$$

The *F*-distribution is formed as the ratio of two chi-square variates divided by their respective degrees of freedom

Density function for the *F*-distribution

$$f(y) = \frac{\Gamma\left[(m+n)/2\right]}{\Gamma(m/2)\Gamma(n/2)}\left(\frac{m}{n}\right)^{m/2}\left(1+\frac{m}{n}y\right)^{(m+n)/2} y^{(m-2)/2}$$

The F-distribution:

- Depends upon two parameters, *m* and *n* which are, respectively the number of degrees of freedom of the chi-square variates in the numerator and denominator.
- Total area under the *F*-curve =1.
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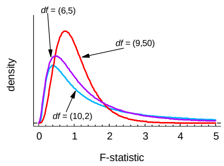
- Has a range from zero to infinity.
- Changes shape as the numbers of the degrees of freedom change.
- As *m* and *n* become large the *F*-distribution has the normal distribution as its limit.
- The mean and variance of the distribution are:

$$\mu = \frac{n}{n-2}, \quad n>2 \qquad \sigma^2 = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}, \quad n>4$$

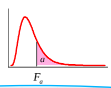
The variance does not exist for *n* less than or equal to 4 and the mean does not exist for *n* less than or equal to 2.

F (variance ratio) Distribution



df = (numerator df, denominator df)

Handout on the F-distribution



Percentage Points of the F-Distribution

		V1							
V2	a	1	2	3	4	5	6	7	8
1	0.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44
	0.050	161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90
	0.025	647.80	799.50	864.20	899.60	921.80	937.10	948.20	956.70
	0.010	4052.00	4999.50	5403.00	5625.00	5764.00	5859.00	5928.00	5982.00
	0.005	16211.00	20000.00	21615.00	22500.00	23056.00	23437.00	23715.00	23925.00
2	0.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37
	0.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37
	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37
	0.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37
	0.005	198.50	199.00	199.20	199.20	199.30	199.30	199.40	199.40
3	0.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25
	0.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85
	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54
	0.010	34.12	30.82	29.46	28.71	28.24	27.91	27.64	27.49
	0.005	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13
4	0.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95
	0.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04
	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98
	0.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80
	0.005	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35

Key Fact: For an *F*-curve with $df = (\nu_1, \nu_2)$, the *F*- value having area α to its left equals the reciprocal of the *F*-value having area α to its right for an *F*-curve with $df = (\nu_2, \nu_1)$

Bolt Diameter in millimeters

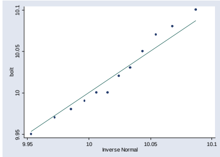
10.05	10.00	10.02	9.97
10.07	10.03	9.98	10.10
9.95	9.99	10.00	10.08

```
. summarize bolt, detail

                            bolt
-------------------------------------------------------------
      Percentiles      Smallest
 1%        9.95           9.95
 5%        9.95           9.97
10%        9.97           9.98       Obs                  12
25%       9.985           9.99       Sum of Wgt.          12

50%       10.01                      Mean              10.02
                        Largest      Std. Dev.      .0469042
75%       10.06          10.05
90%       10.08          10.07       Variance           .0022
95%        10.1          10.08       Skewness        .2815726
99%        10.1           10.1       Kurtosis        1.945364
```

$$s = \sqrt{\frac{\sum_{i=1}^{12}(x_i - \bar{x})^2}{12-1}} \quad (= 0.0469)$$

$$s^2 = \frac{\sum_{i=1}^{12}(x_i - \bar{x})^2}{12-1} \quad (= 0.00220)$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

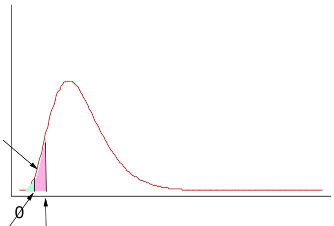


$$\chi^2_{(n-1)} = \frac{n-1}{\sigma_0^2} s^2$$

$$\chi^2_{(n-1)} = \frac{n-1}{\sigma_0^2} s^2 = \frac{11}{0.00810} 0.0022 = 2.988$$

Remember: The Chi-square variate here has *n-1* df because s^2 requires 1 df to compute the sample mean.

$$H_0: \sigma = 0.09\,mm$$
$$H_a: \sigma < 0.09\,mm$$



sdtest bolt = 0.09

```
. sdtest  bolt=0.09

One-sample test of variance

------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    bolt |      12       10.02    .0135401    .0469042    9.990198    10.0498
------------------------------------------------------------------------------

                          Ho: sd(bolt) = 0.09
                           chi2(11) = 2.988

  Ha: sd(bolt) < 0.09         Ha: sd(bolt) != 0.09        Ha: sd(bolt) > 0.09
   P < chi2 = 0.0092          2*(P < chi2) = 0.0182        P > chi2 = 0.9909
```

Elmendorf Tear Strength of Vinyl Floor Coverings

branda	brandb
2288	2592
2368	2512
2528	2576
2144	2176
2160	2304
2384	2384
2304	2432
2240	2112
2208	2288
2112	2752

$$H_0: \sigma_A = \sigma_B$$
$$H_a: \sigma_A \neq \sigma_B$$

$$F = \frac{s_A^2}{s_B^2}$$

Key Fact: Distribution of the F-Statistic for Comparing Two Population Standard Deviations: Suppose that the variable under consideration is normally distributed in each of two populations. Then, for independent samples of sizes n_1 and n_2 from the two populations, the variable

$$F = \frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2}$$

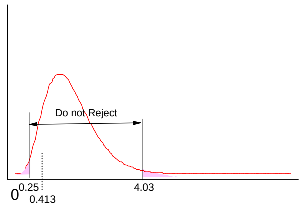
has the F-distribution with df = (n1 -1, n2 -1).

$$F = \frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2} = \frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_A^2} = \frac{s_A^2}{s_B^2}$$

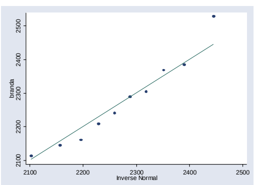
```
. summarize

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      branda |        10      2273.6    128.3218       2112       2528
      brandb |        10      2412.8    199.6691       2112       2752
```

$$F = \frac{s_A^2}{s_B^2} = \frac{128.3^2}{199.7^2} = 0.413$$

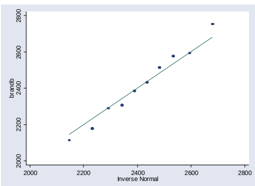
	α			v1						
v2		1	2	3	4	3	6	7	8	9
9	0.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	0.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	0.025	7.21	5,71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	0.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	0.005	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54



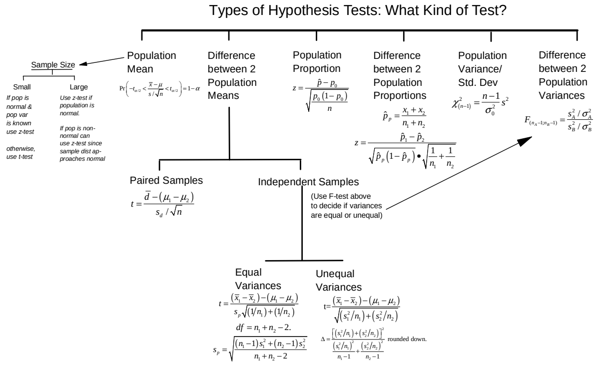
Brand A



Brand B




```
. sdtest branda = brandb

Variance ratio test

------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  branda |      10      2273.6    40.57892    128.3218    2181.804    2365.396
  brandb |      10      2412.8    63.1409     199.6691    2269.965    2555.635
---------+--------------------------------------------------------------------
combined |      20      2343.2    39.86461      178.28    2259.762    2426.638
------------------------------------------------------------------------------

                      Ho: sd(branda) = sd(brandb)

            F(9,9) observed   = F_obs          =    0.413
            F(9,9) lower tail = F_L   = F_obs  =    0.413
            F(9,9) upper tail = F_U   = 1/F_obs =   2.421

    Ha: sd(1) < sd(2)          Ha: sd(1) != sd(2)          Ha: sd(1) > sd(2)
  P < F_obs = 0.1019       P < F_L + P > F_U = 0.2039     P > F_obs = 0.8981
```



Prof. B. A. Turchi